

Non-standard kinetic term as a natural source of non-Gaussianity

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ABSTRACT: We consider reheating after inflation with a non-standard kinetic term. We show that the difference in the kinetic term in different Hubble patches inherited from the long-wavelength moduli inhomogeneities may generate a significant level of non-Gaussianity after inflation.

KEYWORDS: Cosmology of Theories beyond the SM, Superstrings and Heterotic Strings, Supergravity Models, Supersymmetric Effective Theories.

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1. Introduction

Inflation is the main paradigm for understanding the initial conditions for the cosmological perturbations in the early Universe. The observation of the temperature anisotropy of the cosmic microwave background (CMB) supports the standard inflation scenario that predicts a scale-invariant and Gaussian spectrum, except for some anomalies. An obvious signature of such an anomaly is the observation of a spectrum index $n \neq 1$, which suggests that the spectrum is not exactly scale-invariant [1]. The observation of $n \neq 1$ is important because it distinguishes several inflationary models. In this paper, we consider another signature of inflation: non-Gaussianity in the spectrum [2, 3]. In fact, some inflationary models predict some level of non-Gaussianity in the spectrum. For example, generation of non-Gaussian perturbations may occur (1) during inflation by a step-like potential [4], (2) during inflation by a kinetic term [5], (3) during inflation by a modulated velocity [6, 7], (4) at the end of inflation [8–10], (5) after inflation at (p)reheating [11–14], and (6) long after inflation [15, 17–19]. Except for cases (1) and (2), long-wavelength inhomogeneities of a light field give rise to non-Gaussianity. It is important to consider models of inflation in which some level of non-Gaussianity follows when a curvature perturbation is generated. It would also be interesting to consider possibilities for adding non-Gaussian perturbations at some event in the early Universe, if it appears as a natural consequence of basic properties of the model. In this paper, we consider reheating after inflation with a non-standard kinetic term. Reheating is a common feature of the inflationary scenario, and a non-standard kinetic term arises naturally in some string inspired models and also in other extended models of general gravity.

2. The model

We consider preheating in a theory of two interacting scalar fields $\{\phi, \chi\}$ with moduli σ with non-canonical kinetic term:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_p^2 \mathcal{R} - \frac{\omega(\sigma)}{2} (\partial_\mu \phi) (\partial^\mu \phi) - \frac{1}{2} (\partial_\mu \chi) (\partial^\mu \chi) - \frac{1}{2} (\partial_\mu \sigma) (\partial^\mu \sigma) - V(\phi, \chi, \sigma), \right] \quad (2.1)$$

where M_p is the reduced Planck mass and the potential $V(\phi, \chi, \sigma)$ is given by

$$V(\phi, \chi, \sigma) = \frac{1}{2}m_\phi\phi^2 + \frac{1}{2}m_\chi\chi^2 + \frac{g^2}{2}\phi^2\chi^2 + W(\sigma). \quad (2.2)$$

We assume for simplicity that ϕ is the inflaton field, which starts oscillating about the minimum after inflation, and χ is the preheat field whose perturbations $\delta\chi$ grow rapidly near the enhanced symmetric point (ESP). Inflation ends and oscillation starts when $\phi = \phi_0$, which gives the initial amplitude of the ϕ -oscillation. The background fields evolve according to the equations

$$\ddot{\phi} + 3H\dot{\phi} + \frac{(m_\phi^2 + g^2\chi^2)\phi}{\omega} + \frac{\omega_\sigma}{\omega}\dot{\phi}\dot{\sigma} = 0 \quad (2.3)$$

$$\ddot{\chi} + 3H\dot{\chi} + (m_\chi^2 + g^2\phi^2)\chi = 0 \quad (2.4)$$

$$\ddot{\sigma} + 3H\dot{\sigma} + W_\sigma - \frac{1}{2}\omega_\sigma\dot{\phi}^2 = 0, \quad (2.5)$$

where the subscripts for ω and W indicate derivatives with respect to the fields. The Hubble parameter is given by

$$H^2 \equiv \frac{1}{3M_p^2} \left[\frac{1}{2}\omega\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}\dot{\sigma}^2 + V \right]. \quad (2.6)$$

For the instant preheating scenario, the velocity of the oscillating field at the bottom of the potential determines the number density of the preheat field. For a standard kinetic term (i.e., for $\omega \equiv 1$), the velocity of the oscillating field is given by

$$\dot{\phi}_{\max} \simeq m_\phi\phi_0, \quad (2.7)$$

while for a non-standard kinetic term, it is given by

$$\dot{\phi}_{\max} \simeq \frac{m_\phi\phi_0}{\sqrt{\omega}}. \quad (2.8)$$

Therefore, $\dot{\phi}_{\max}$ at the bottom of the potential is different in different Hubble patches if the perturbation $\delta\omega$ is inherited from the long-wavelength moduli inhomogeneities $\delta\sigma$. The condition for the perturbations of σ to cross the horizon during inflation is

$$\eta_\sigma^{\text{eff}} = M_p^2 \left[\frac{W_{\sigma\sigma} - \omega_\sigma\dot{\phi}^2/2}{V} \right] \simeq M_p^2 \left[\frac{W_{\sigma\sigma}}{V} - \frac{1}{2V} \frac{\omega_\sigma}{\omega^2} \left(\frac{m_\phi^2\phi}{3H} \right)^2 \right] \sim -\eta_\phi^2 \frac{\omega_\sigma\dot{\phi}^2}{\omega^2} \ll 1, \quad (2.9)$$

where $W_{\sigma\sigma}/3H^2 \ll 1$ is assumed. σ moves slowly during inflation if the following slow-roll condition is satisfied:

$$\epsilon_\sigma^{\text{eff}} = \frac{M_p^2}{2} \left[\frac{W_\sigma - \omega_\sigma\dot{\phi}^2/2}{V} \right]^2 \sim \eta_\phi^4 \omega_\sigma^2 \dot{\phi}^2 \left(\frac{\phi^2}{M_p^2} \right) \ll 1, \quad (2.10)$$

where $\frac{M_p^2}{2} \left(\frac{W_\sigma}{V} \right)^2 \ll 1$ is assumed. To understand these conditions, we consider a specific choice of ω :

$$\omega = e^{\alpha \frac{\sigma^2}{M_*^2}}, \quad (2.11)$$

where α is a dimensionless constant and M_* is the cut-off scale of the effective theory. We find from eq. (2.9) that

$$-4\alpha^2 \frac{\sigma^2}{M_*^2} - 2\alpha \ll \frac{M_*^2 \omega}{\eta_\phi^2 \phi^2}. \quad (2.12)$$

Note that the field σ has negative η_σ^{eff} for $\alpha > 0$. The condition from the effective ϵ -parameter leads to a similar condition

$$4\alpha^2 \frac{\sigma^2}{M_*^2} \ll \frac{M_*^2 M_p^2}{\omega^2 \eta_\phi^4 \phi^4}. \quad (2.13)$$

Following these conditions, we assume a natural condition $\sigma \ll M_*$ and consider a slow-rolling σ field during inflation.

Considering the reheating process after inflation, the inflaton field must finally decay into the Standard-Model (SM) particles. Here we consider the primary decay process of the inflaton field: $\phi \rightarrow \chi$. Since the interaction depends on the values of the fields ϕ and χ , the background field trajectories will be very sensitive to the initial conditions and the non-perturbative effects of the preheating process, which means that the general evaluation of the cosmological parameters typically requires numerical calculations. In this paper, we consider the instant preheating scenario [20] so that an analytic estimate can be made for the non-linear parameter for the CMB spectrum. Applying the results of ref. [20], the comoving number density of the preheat χ particles produced at the ESP during the first half-oscillation is

$$n_\chi = \frac{(g|\dot{\phi}_{\text{max}}|)^{3/2}}{8\pi} \exp\left\{-\frac{\pi g|\phi_{\text{min}}|^2}{|\dot{\phi}_{\text{max}}|}\right\} \simeq \frac{(g|\dot{\phi}_{\text{max}}|)^{3/2}}{8\pi}. \quad (2.14)$$

Due to the interaction term, the effective mass of the preheat field increases as the oscillating field moves away from the bottom of the potential. Thus, the preheat particles produced soon acquire large mass and decay into ψ particles with a decay rate Γ_χ . Depending on the couplings between particles, the decay process from ϕ to ψ is so fast that all the energy stored in the oscillating field may turn into ψ particles. The ψ particles then thermalize to complete the reheating process. To determine the curvature perturbations produced during the reheating process, it is important to consider the relation $\rho_\chi \propto n_\chi$.¹ Then, we obtain the curvature perturbation ζ generated during preheating:

$$\zeta \equiv -\psi - H \frac{\delta\rho_\chi}{\dot{\rho}_\chi} \simeq \beta \frac{\delta n_\chi}{n_\chi}, \quad (2.15)$$

where β is a proportionality constant that depends on the redshifting of the preheat particles, and in the last step we considered a spatially flat gauge. To obtain an analytic

¹Here we followed the first paper in ref. [11] and assumed instant decay (i.e., the preheat particles decay at $m_\chi \simeq g\phi_c$ before ϕ turns around. It is possible to construct inhomogeneous preheating models without the instant decay. See for example ref. [16].

estimate of the curvature perturbation, from eq. (2.14) we find

$$\begin{aligned} \frac{\delta n_\chi}{n_\chi} &= \left[\frac{3}{2} + \frac{\pi g |\phi_{\min}|^2}{|\dot{\phi}_{\max}|} \right] \frac{\delta |\dot{\phi}_{\max}|}{|\dot{\phi}_{\max}|} - \frac{2\pi g |\phi_{\min}|^2}{|\dot{\phi}_{\max}|} \frac{\delta |\phi_{\min}|}{|\phi_{\min}|} \\ &\simeq \frac{3}{2} \frac{\delta |\dot{\phi}_{\max}|}{|\dot{\phi}_{\max}|}. \end{aligned} \quad (2.16)$$

Here, $\dot{\phi}_{\max}$ is given by eq. (2.8), and the long-wavelength inhomogeneities of σ causes $\delta\omega$. According to ref. [11], multi-field inflation may lead to a significant $\delta\phi_{\min}$ and to $\phi_{\min} \neq 0$ at the minimum of the oscillation, which causes significant inhomogeneities of n_χ . However, considering single-field or symmetric multi-field potential, the minimum of the oscillation trajectory is at $\phi = 0$, and hence we may disregard terms proportional to $\phi_{\min} = 0$. The long-wavelength inhomogeneities of $\delta\omega$ then lead to fluctuations of $\dot{\phi}_{\max}$:

$$\delta\dot{\phi}_{\max} \simeq -\frac{1}{2} \frac{\delta\omega}{\omega} \dot{\phi}_{\max}, \quad (2.17)$$

which eventually gives the curvature perturbation:

$$\zeta \simeq \beta \frac{\delta n_\chi}{n_\chi} \simeq \frac{3\beta}{4} \frac{\delta|\omega|}{|\omega|}. \quad (2.18)$$

For the specific choice of ω given by eq. (2.11), we find

$$\frac{\delta\omega}{\omega} \simeq \frac{\omega_\sigma}{\omega} \delta\sigma + \frac{1}{2} \frac{\omega_{\sigma\sigma}}{\omega} (\delta\sigma)^2 \simeq \alpha \frac{2\sigma}{M_*} \frac{\delta\sigma}{M_*} + \left[2\alpha^2 \frac{\sigma^2}{M_*^2} + \alpha \right] \frac{(\delta\sigma)^2}{M_*^2} \quad (2.19)$$

and the curvature perturbation is then

$$\zeta \simeq \frac{3\beta|\alpha|}{4} \left| \frac{2\sigma}{M_*} \frac{\delta\sigma}{M_*} + \left\{ 2\alpha \frac{\sigma^2}{M_*^2} + 1 \right\} \frac{(\delta\sigma)^2}{M_*^2} \right|. \quad (2.20)$$

The level of the non-Gaussianity is specified by the non-linear parameter f_{NL} , which is defined by the Bardeen potential Φ :

$$\Phi = \Phi_{\text{Gaussian}} + f_{\text{NL}} \Phi_{\text{Gaussian}}^2, \quad (2.21)$$

which is connected to the curvature perturbation ζ through

$$\Phi = -\frac{3}{5} \zeta. \quad (2.22)$$

If the first-order perturbation is dominantly generated by the usual inflaton perturbation, the second-order perturbation that is generated by the long-wavelength moduli inhomogeneities is not correlated to the first-order perturbation. In this case, the estimate of the non-linear parameter is given by [17]:

$$\frac{6}{5} f_{\text{NL}} \simeq \frac{1}{N_\phi^4} \left[N_\sigma^2 N_{\sigma\sigma} + N_{\sigma\sigma}^3 \mathcal{P}_\sigma \log(k_b L) \right], \quad (2.23)$$

where the curvature perturbation ζ is expanded by the δN formalism as

$$\zeta \simeq N_\phi \delta\phi + N_\sigma \delta\sigma + \frac{1}{2} N_{\phi\phi} \delta\phi^2 + \frac{1}{2} N_{\sigma\sigma} \delta\sigma^2 + \dots, \quad (2.24)$$

and the perturbation can be separated into two parts:

$$\zeta = \zeta^{(\phi)} + \zeta^{(\sigma)}. \quad (2.25)$$

Here $k_b \equiv \min\{k_i\}$ ($i = 1, 2, 3$) is the minimum wavevector of the bispectrum and L is the size of a box in which the perturbation is defined. The equation for the non-linear parameter can be simplified to obtain [21]

$$f_{\text{NL}} \simeq \left(\frac{1}{1300} \frac{N_{\sigma\sigma}}{N_\phi^2} \right)^3. \quad (2.26)$$

Assuming $\sigma \ll M_*$ at reheating, we obtain

$$f_{\text{NL}} \simeq \left(10^6 \times \alpha\beta \frac{H^2}{M_*^2} \right)^3, \quad (2.27)$$

which becomes large as H approaches the cut-off scale M_* . Although it depends on the parameters of the model, the result is very interesting since the upper bound for f_{NL} may apply a significant upper bound on the inflation energy scale. The calculation suggests that the observation of the non-linear parameter f_{NL} may allow the inflation energy scale to be determined, dependent on the model parameters determined by some other experiments.

3. Conclusions and discussions

In this paper, we considered reheating after inflation with a non-standard kinetic term. We have shown that the difference in the kinetic term in different Hubble patches inherited from the long-wavelength moduli inhomogeneities $\delta\sigma$ causes generation of a significant level of non-Gaussianity after inflation.

For completeness, we consider the tachyonic evolution of the light field σ during the oscillating phase. During the half-oscillation the oscillating field ϕ follows $\dot{\phi}(t) = -\frac{m_\phi}{\sqrt{\omega}}\phi_0 \sin \frac{m_\phi}{\sqrt{\omega}}t$, which leads to the equation of motion for the light field σ :

$$\ddot{\sigma} + 3H\dot{\sigma} + W_\sigma - \frac{1}{2}\omega_\sigma \frac{m_\phi^2}{\omega} \phi_0^2 \sin^2 m_\phi t = 0. \quad (3.1)$$

Omitting the term proportional to $\dot{\sigma}$ and disregarding the variation of ω , the equation can be reduced to the well-known Mathieu equation for the specific choice of ω given by eq. (2.11). However, comparing the above equation with the standard preheating equation for the original model [20], the coupling g in the standard interaction term $g^2\phi^2\sigma^2/2$ is replaced by an effective coupling

$$g_{\text{eff}} \sim \alpha \frac{m_\phi^2}{M_*^2 \omega}, \quad (3.2)$$

which is very small if $m_\phi \ll M_*$. We can also find a small slow-roll parameter $\epsilon_\sigma^{\text{eff}} \ll 1$ for $\sigma \ll M_*$. Therefore, in this case the light field σ is also slow-rolling in the oscillating phase. Using a similar analysis for the initial condition $\sigma \ll M_*$, we find that the fluctuation of the light field $\delta\sigma$ does not evolve exponentially during the oscillating phase, even if η_σ^{eff} is negative (i.e., tachyonic) during this phase. Besides the possibility of exponential growth of the long-wavelength fluctuation $\delta\sigma$, which may take place for $\sigma \sim M_*$ and $\eta_\sigma^{\text{eff}} < 0$, it is possible to consider a more generic form of the effective action. In this paper, we considered a simple form of the potential assuming a minimum at $\phi_{\text{min}} = 0$. However, in more generic cases we may consider a symmetry breaking potential where the global minimum of the potential is not at the origin. Recently, preheating with a non-standard kinetic term and with a symmetry breaking potential has been studied by Lachapelle et al. [22], in which the oscillating field determines the coefficient ω . It has been shown that the non-standard kinetic term may give rise to a more efficient preheating channel than the usual process caused by the standard interaction term. It is possible to extend our arguments to more generic actions based on some specific string models containing non-standard kinetic terms that depend on several moduli fields. We can also extend our analysis to more generic initial conditions. In such cases, non-standard kinetic terms may give rise to more efficient preheating than the standard interaction, as discussed by Lachapelle et al., or they may cause tachyonic enhancement of $\delta\sigma$ during oscillation. These effects may lead to a more significant level of non-Gaussianity and to a more effective constraint for the effective action. These complementary scenarios requires numerical study, hence they will be considered in future works.

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